Chapter 5

Log and Exponential Functions

Review Summary

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Sect 5-1: The Natural log function, Differentiation

1) $\log(A \cdot B) = \log A + \log B$

2) $\log\left(\frac{A}{B}\right) = \log A - \log B$

* 3) $\log A^c = c \log A$

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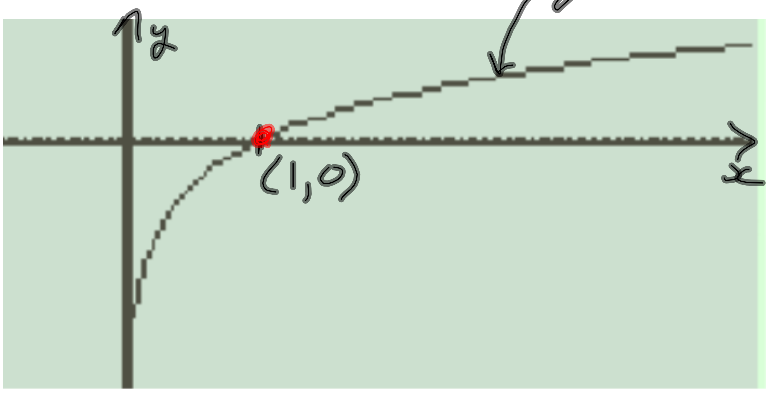
Theorem 5-1: Properties of Natural Logarithmic Functions

1) The Domain is $(0, \infty)$ and the range is $(-\infty, \infty)$

2) The function is continuous, increasing and one-to-one $\Leftrightarrow$ horizontal line test

3) The graph is concave downward.

$y = \ln x$



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Theorem 5-3:

Derivative of the Natural Logarithmic Function

$$\boxed{1)\ \frac{d}{dx}\ln x = \frac{1}{x} \quad \text{for } x > 0}$$

$$\boxed{2)\ \frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}}$$

Chain Rule

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Section 5-2

The Natural Logarithmic Function

Integration

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Theorem 5-5

Log Rule for integration:

$$\boxed{\int \frac{1}{x}\,dx = \ln|x| + c}$$

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*Will be given to you on Final

Integrals of Trig Functions

1) $\int \tan u\,du = -\ln|\cos u| + c$

2) $\int \cot u\,du = \ln|\sin u| + c$

3) $\int \sec u\,du = \ln|\sec u + \tan u| + c$

4) $\int \csc u\,du = -\ln|\csc u + \cot u| + c$

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Average Value

$$\boxed{\text{Ave VALUE} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

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Section 5-3

Inverse Functions

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Definition of Inverse Functions

g is the inverse of f if:

$\boxed{f(g(x)) = x}$

for each x in the domain of g.

$\boxed{g(f(x)) = x}$

for each x in the domain of f.

* The inverse is also denoted $f^{-1}(x)$

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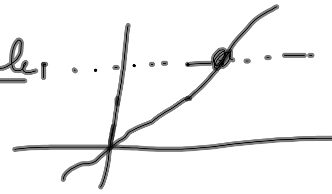
Horizontal Line Test

A function is one-one if every horizontal line intersects the graph at one point.

Example:



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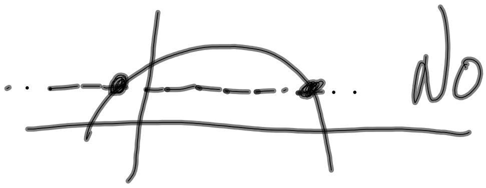

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Theorem 5-7

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The Existence of an Inverse Function

1) A function has an inverse function if and only if it is ONE-ONE.

2) If f is strictly monotonic (increasing or decreasing) on its entire domain, then it is one-one and therefore has an inverse (use FDT)

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Theorem 5-9

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The Derivative of an Inverse Function

Let f be a function that is differentiable on I.

If f has an inverse g, then $f'(g(x)) \neq 0$ and

$$\boxed{g'(x) = \frac{1}{f'(g(x))}}$$

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Derivative of inverse function

$$f(f^{-1}(x)) = x$$

Take derivative

$$f'(f^{-1}(x)) \cdot [f^{-1}(x)]' = 1$$

$$\boxed{[f^{-1}(x)]' = \frac{1}{f'[f^{-1}(x)]}}$$

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Derivative of inverse function

$$f[f^{-1}(x)] = x$$

Take the derivative:

$$f'[f^{-1}(x)] \cdot [f^{-1}(x)]' = 1$$

$$\boxed{[f^{-1}(x)]' = \frac{1}{f'[f^{-1}(x)]}}$$

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Section 5-4

Exponential Functions: Differentiation and Integration

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Definition of the Natural Exponential Function

$$\boxed{f(x) = \ln x}$$

$\rightarrow \log_e x$

$$\boxed{f^{-1}(x) = e^x}$$

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Derivative of the Natural Exponential Function

1) $\frac{d}{dx}[e^x] = e^x$

2) $\frac{d}{dx}[e^u] = e^u \cdot \frac{du}{dx}$

Proof

$\ln e^x = x$

$\frac{d}{dx}[\ln e^x] = \frac{d}{dx} x$

$\downarrow$

chain rule

$\Rightarrow \frac{1}{e^x} \cdot \frac{d}{dx} e^x = 1$

$$\boxed{\frac{d}{dx} e^x = e^x}$$

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Theorem 5.12

Integration Rules for Exponential Functions

$$\boxed{\int e^x\,dx = e^x + c}$$

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Section 5-5:

Bases other than e

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Definition of a Log function to base a

$$\boxed{\log_a x = \frac{\ln x}{\ln a}}$$

Change of Base formula

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Properties of Inverse Functions

1) $y = a^x \Longleftrightarrow x = \log_a y$

2) $a^{\log_a x} = x$ for $x > 0$

3) $\log_a a^x = x$ for all x

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Theorem 5-13:

Derivatives for Bases other than e

Let a be a positive real number and $a \neq 1$.

$$\boxed{1)\ \frac{d}{dx} a^x = a^x \ln a}$$

$$\boxed{2)\ \frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}}$$

$$\boxed{3)\ \frac{d}{dx} \log_a x = \frac{1}{x \ln a}}$$

$$\boxed{4)\ \frac{d}{dx} \log_a u = \frac{1}{u \ln a} \cdot \frac{du}{dx}}$$

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Integral of A Base other than e

$$\int a^x dx = \boxed{\frac{a^x}{\ln a} + C}$$

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Section 5-6: I

Inverse Trigonometric Functions: Differentiation

Why its important:

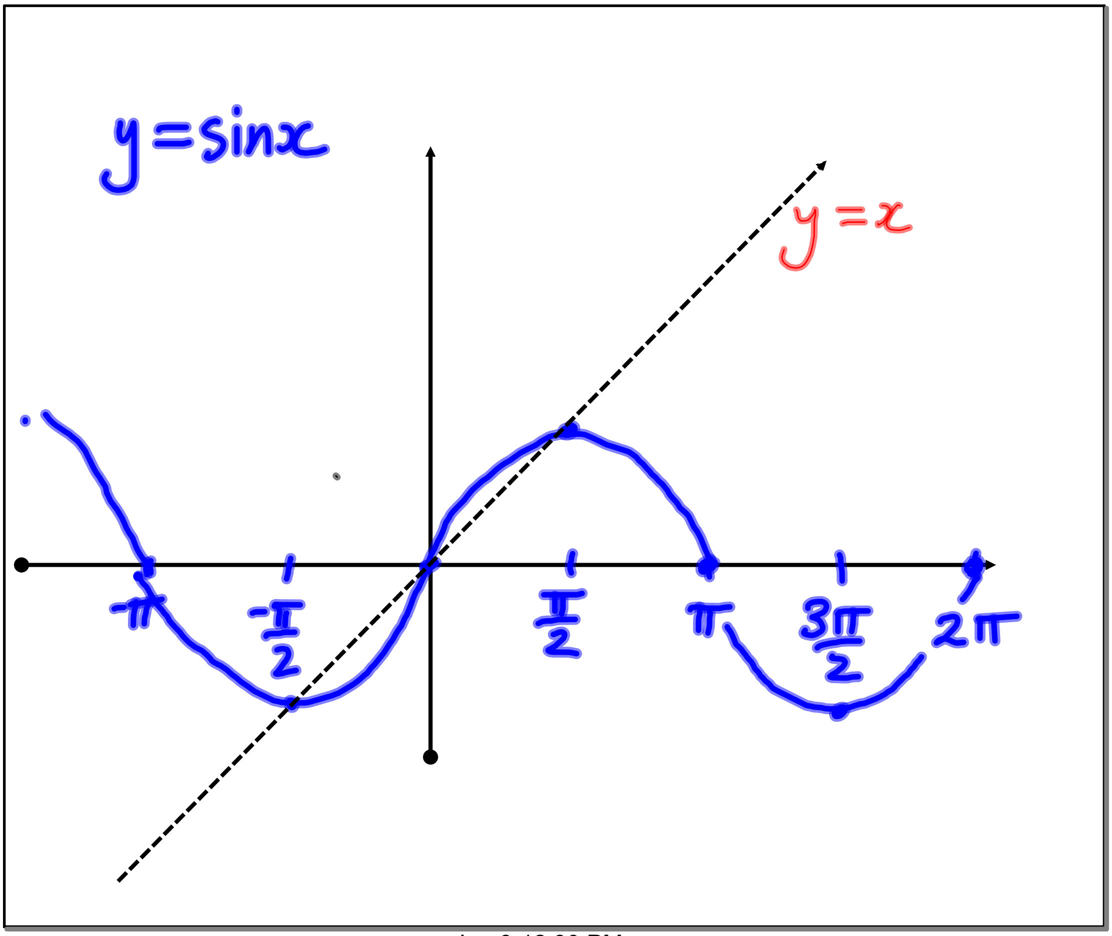
- Derivative of inverse trig shows up in Economics, Biology, Chemistry and Physics AND its on the AP Calculus BC exam

$y = \arccos x$

↳ inverse of $\cos x$

written $y = \cos^{-1} x$.

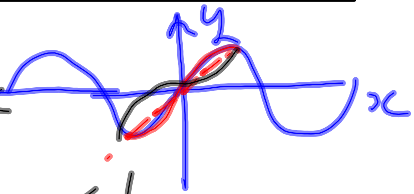
Also * $y = \cos^{-1} x$ is not the same as $y = \frac{1}{\cos x} = \sec x$!



Definitions of Inverse Trig Functions

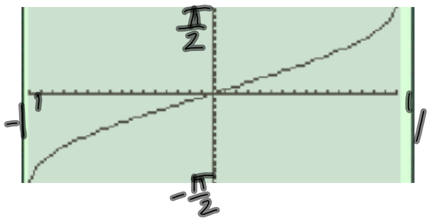
CALC $\Rightarrow \sin^{-1}x$

1) $y = \arcsin x$



Domain: $-1 \le x \le 1$

Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



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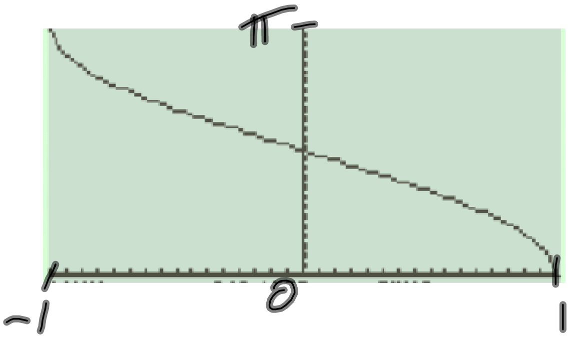
2) $y = \arccos x$

Domain: $-1 \le x \le 1$

Range: $0 \le y \le \pi$



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Theorem 5-16

Derivatives of Inverse Trig Functions

1) $\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$

2) $\frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$

3) $\frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$

4) $\frac{d}{dx}\operatorname{arc}\cot u = \frac{-u'}{1+u^2}$

5) $\frac{d}{dx}\operatorname{arc}\sec u = \frac{u'}{|u|\sqrt{u^2-1}}$

6) $\frac{d}{dx}\operatorname{arc}\csc u = \frac{-u'}{|u|\sqrt{u^2-1}}$

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Section 5-7

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Inverse Trig Functions: Integration

NOTE: $a>0$

1) $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$

2) $\int \frac{du}{\sqrt{a^2-u^2}} = \arccos\frac{u}{a} + C$

3) $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

4) $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arc}\sec\frac{|u|}{a} + C$

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