Section 5-3

Inverse Functions

TEST: Friday, Sept. 26th

Definition of Inverse Functions

If $\boxed{f(g(x)) = x}$

and

$\boxed{g(f(x)) = x}$

Then $g(x)$ is the inverse of $f(x)$

NOTE:

The inverse is sometimes written $f^{-1}(x)$ however it is **not** the same as

$$\frac{1}{f(x)}$$

Example: $y = \ln x$

$g(x) = e^{x}$

$\ln(e^{x}) = x$

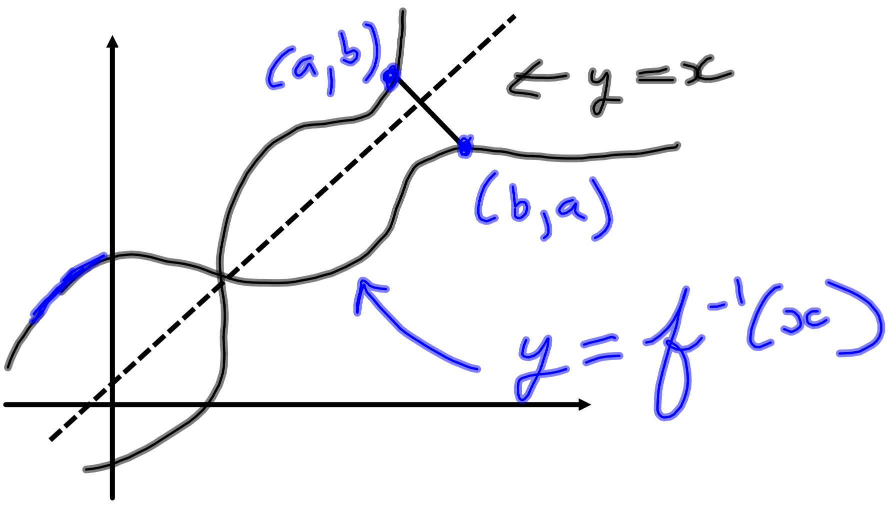
$e^{\ln x} = x$

These are inverses!

Theorem 5-6

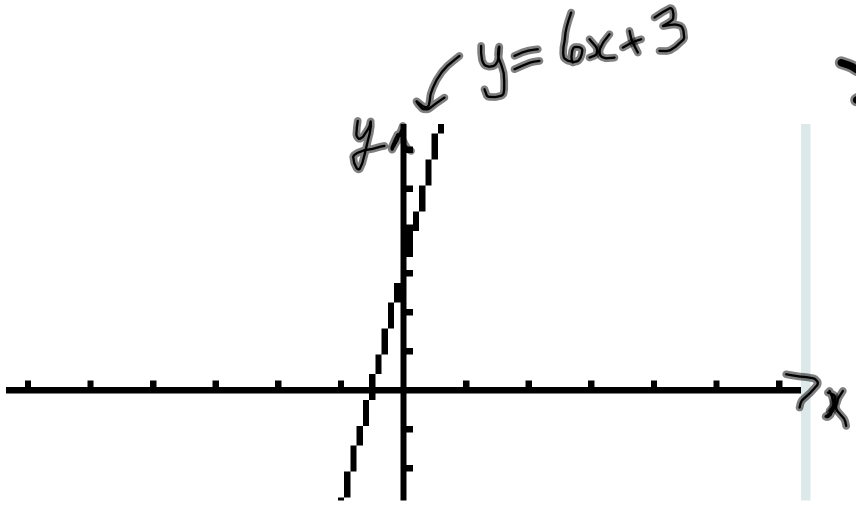
Reflective Property of Inverse Functions

The graph of f contains thepoint(a,b) iff the graph of f inverse contains the point (b,a)



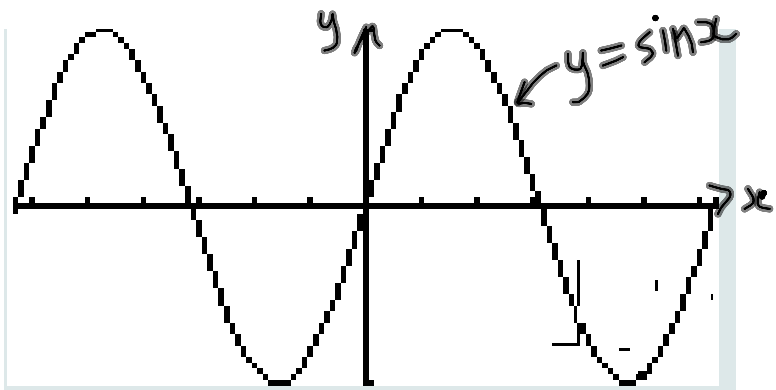
Horizontal Line Test

A function is one-one if every horizontal line intersects the graph at one point.



Yes

No



Theorem 5-7

The Existence of an Inverse Function

1) A function has an inverse iff it is ONE-ONE

2) If f is strictly Monotonic (always increasing or always decreasing) on its entire domain,then it must be one-one and therefore has an inverse. We may use FDT to test this.

Guidelines for Finding an Inverse Function

1) Check to see if its one-one.
2) Get y alone.
3) Switch x and y.
4) Solve for the "new" y.
5) Call it $f^{-1}(x)$

Theorem 5-8:

Continuity and Differentiability of Inverse Functions

1) If f is continuous on its domain,then its inverse is continuous.

2) If f is decreasing, the its inverse must also be decreasing. (same is true for increasing)

3) If f is differentiable on an interval containing c and f'(c) is not equal to 0, then f^{-1} is also differentiable. at $f(c)$.

Theorem 5-9

The Derivative of an Inverse Function

Let f be a function that is differentiable on I. If f has an inverse g then $f'(g(x)) \neq 0$ and

$$\boxed{g'(x) = \frac{1}{f'(g(x))}}$$

Proof:

$$f(f^{-1}(x)) = x$$

$$\frac{d}{dx} f(f^{-1}(x)) = \frac{d}{dx} x$$

Chain Rule:

$$f'[f^{-1}(x)] \cdot [f^{-1}(x)]' = 1$$

$$[f^{-1}(x)]' = \frac{1}{f'[f^{-1}(x)]}$$

or $$[g(x)]' = \frac{1}{f'(g(x))}$$

Examples: Page 350. SHOW that f and g are inverses

a) Algebraically b) Graphically

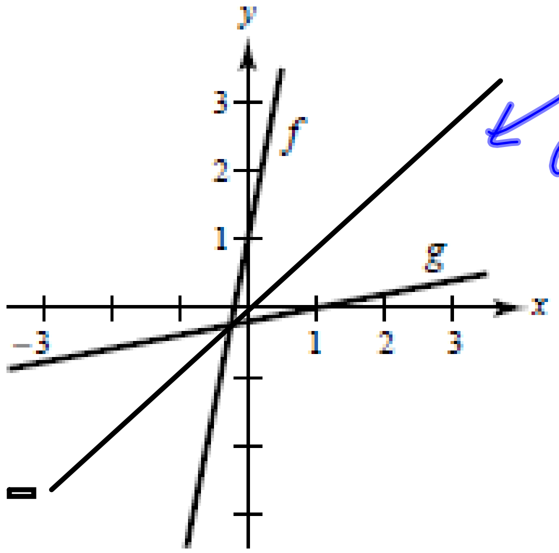
1. (a) $f(x) = 5x + 1$

$$g(x) = \frac{x-1}{5} \quad \text{or } f^{-1}(x)$$

$$f(g(x)) = f\left(\frac{x-1}{5}\right) = 5\left(\frac{x-1}{5}\right) + 1 = x$$

$$g((f(x)) = g(5x + 1) = \frac{(5x+1) - 1}{5} = x$$

(b)



$y = x$

a) Find the inverse.
b) Graph f and f^(-1).
c) Write Domain.

D: of f and f^{-1}
all reals

R: of f, f^{-1}
all reals

25) $f(x) = x^5$

$y = x^5$

$x = y^5$

$y = x^{1/5}$

$f^{-1}(x) = x^{1/5}$

stop here 09/18

Use the Derivative to determine whether the function is strictly monotonic on its entire domain and therefore has an inverse.

41) $f(x) = 2 - x - x^3$

$f'(x) = -1 - 3x^2$

$0 = -1 - 3x^2$

$3x^2 = -1$

$x^2 = -\frac{1}{3}$

Strictly monotonic, has inverse.

Use the derivative to determine whether the function is monotonic

44) $f(x) = \frac{x^4}{4} - 2x^2$

Critical pts

$f'(x) = x^3 - 4x$

$0 = x(x^2 - 4)$

C.N.'s $x = 0$, $x = \pm 2$

x	-3	-2	-1	0	1	2	3
$f'(x)$	$-$	rel min	$+$	rel max	$-$	rel min	$+$

NOT MONOTONIC

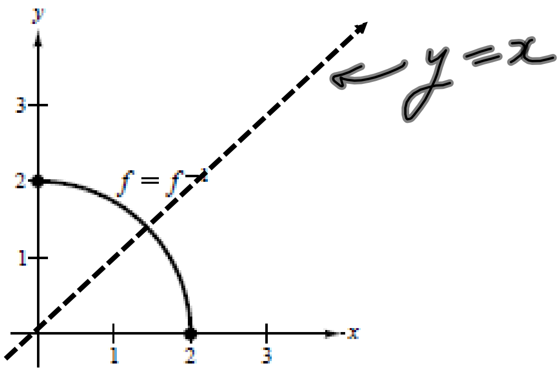
Therefore there is NO INVERSE

35. $f(x) = \sqrt{4 - x^2} = y, \quad 0 \le x \le 2$

$$x = \sqrt{4 - y^2}$$

$$y = \sqrt{4 - x^2}$$

$$f^{-1}(x) = \sqrt{4 - x^2}, \; 0 \le x \le 2$$



Verify that f has an inverse.
Then use the function f and a
to find f^(-1) at a.

7) $f(x) = x^3 - 1$, $a = 26$

ONE-ONE function.

$y = x^3 - 1$ $\qquad f'(x) = 3x^2$

$x = y^3 - 1$

$x + 1 = y^3$

$y = \sqrt[3]{x+1}$

$f^{-1}(x) = \sqrt[3]{x+1} = g(x)$

$$g'(x) = \frac{1}{f'(g(x))}$$

$$g'(x) = \frac{1}{3\left(\sqrt[3]{x+1}\right)^2}$$

$$= \frac{1}{3\left(\sqrt[3]{27}\right)^2}$$

$$= \frac{1}{3(9)} = \boxed{\frac{1}{27}}$$

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#2,4,24- 32e,42,72,74,76

Electric Version CHANGE:

28->27,
30->29,