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#4-28 evens

4) $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$

$$\underset{a_n}{\frac{1}{3n^2+2}} < \underset{b_n}{\frac{1}{3n^2}}$$

and: $\frac{1}{3}\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series with $p=2>1$)

$\therefore$ Since $\sum b_n$ converges (p-series)
$\sum a_n$ converges by D.C.T.

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6) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

$b_n \qquad a_n$

$\frac{1}{\sqrt{n}-1} > \frac{1}{\sqrt{n}}$ ← p-series with $p=\frac{1}{2}$ so Divergent

Since $\sum a_n$ diverges

$\Rightarrow \sum b_n$ also diverges

by D.C.T.

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8) $\sum_{n=0}^{\infty} \frac{4^n}{5^n+3}$

$a_n \qquad b_n$

$$\frac{4^n}{5^n+3} < \left(\frac{4}{5}\right)^n$$

geometric series with $r = \frac{4}{5} < 1$ converges.

$\therefore$ Since $\sum b_n$ converges

$\sum a_n$ converges by D.C.T.

10) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$

$$\underbrace{\frac{1}{\sqrt{n^3+1}}}_{a_n} < \underbrace{\frac{1}{n^{3/2}}}_{b_n}$$

Convergent p-series with $p = 3/2 > 1$

Since $\sum b_n$ converges

$\Rightarrow \sum a_n$ converges by D.C.T.

12) $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$

$$\overset{b_n}{\frac{1}{4\sqrt[3]{n}-1}} > \overset{a_n}{\frac{1}{4\sqrt[3]{n}}}$$

$\hookrightarrow \frac{1}{4} \cdot \frac{1}{n^{1/3}}$

$\downarrow$

Divergent p-series.

Since $\sum a_n$ diverges

$\Rightarrow$ $\sum b_n$ also diverges

by D.C.T.

14) $\sum_{n=1}^{\infty} \frac{3^n}{2^n-1}$

$\underbrace{\frac{3^n}{2^n-1}}_{b_n} > \underbrace{\left(\frac{4}{3}\right)^n}_{a_n}$

$\hookrightarrow$ geometric with $r = \frac{4}{3} > 1$
$\therefore$ diverges.

Since $\sum a_n$ diverges
$\Rightarrow \sum b_n$ diverges by D.C.T.

16) $\sum_{n=1}^{\infty} \frac{5}{4^n+1}$

$$\lim_{n\to\infty} \frac{\frac{5}{4^n+1}}{\frac{1}{4^n}}$$

a_n

b_n convergent geometric series $r=\frac{1}{4}<1$

$$= \lim_{n\to\infty} \frac{4^n \cdot 5}{4^n+1} \quad \begin{matrix}\to\infty \\ \to\infty\end{matrix}$$

$\frac{d}{dx}a^x = a^x \ln a$

L'Hop

$$= \lim_{n\to\infty} \frac{4^n \cdot \ln 4 \cdot 5}{4^n \ln 4} = 5$$

↳ pos, finite

Since $\sum b_n$ converges

$\sum a_n$ also converges by L.C.T.

8) $\sum_{n=1}^{\infty} \frac{2^n+1}{5^n+1}$

$$\lim_{n\to\infty} \frac{\frac{2^n+1}{5^n+1}}{\left(\frac{2}{5}\right)^n} \quad \begin{matrix} a_n \\ b_n \end{matrix}$$

↳ convergent Geometric Series. $(r = \frac{2}{5} < 1)$

$$= \lim_{n\to\infty} \frac{2^n 5^n + 5^n}{5^n 2^n + 2^n}$$

Divide top and bottom by $2^n 5^n$

$$= \lim_{n\to\infty} \frac{1 + \frac{1}{2^n}}{1 + \frac{1}{5^n}} = 1$$

positive, finite

Since $\sum b_n$ converges

$\Rightarrow \sum a_n$ also converges by the L.C.T.

20) $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{n^3-2n+3}$

$$\lim_{n\to\infty} \frac{\dfrac{n+5}{n^3-2n+3}}{\dfrac{1}{n^2}} \quad \begin{matrix} a_n \\ \\ b_n \end{matrix}$$

convergent p-series

$$= \lim_{n\to\infty} \frac{n^3+5n^2}{n^3-2n+3} = 1$$

pos, finite

use $y = a/c$ where a & c are the leading coefficients

OR use L'Hopital's rule

Since $\sum b_n$ converges then

$\sum a_n$ also converges

by L.C.T.

22) $$\sum_{n=1}^{\infty} \frac{1}{n^2(n+3)}$$

$$\lim_{n\to\infty} \frac{\dfrac{1}{n^2(n+3)}\ a_n}{\dfrac{1}{n^3}\ b_n}$$

$\dfrac{1}{n^3}$ → Convergent p-series

$$\lim_{n\to\infty} \frac{n}{(n+3)} = 1$$

→ pos, finite.

Since $\sum b_n$ converges

$\Rightarrow \sum a_n$ converges by L.C.T.

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24) $\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n-1}}$

$= \lim_{n\to\infty} \frac{\frac{n}{(n+1)2^{n-1}} \; a_n}{\left(\frac{1}{2}\right)^n \; b_n}$ convergent $(r=\frac{1}{2})$ geometric series

$= \lim_{n\to\infty} \frac{n\, 2\, 2^n}{(n+1)2^n} = 2$ → pos + finite

Since $\sum b_n$ converges

$\Rightarrow \sum a_n$ converges by L.C.T.

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26) $\sum_{n=1}^{\infty} \frac{5}{n+\sqrt{n^2+4}}$

$$= \lim_{n\to\infty} \frac{\frac{5}{n+\sqrt{n^2+4}}}{\frac{1}{n}} \quad \begin{matrix} a_n \\ b_n \end{matrix}$$

Divergent Harmonic series

$$= \lim_{n\to\infty} \frac{5n}{n+(n^2+4)^{1/2}}$$

*Don't use L'Hop here. It is much easier to divide everything by n.

$$= \lim_{n\to\infty} \frac{\frac{5n}{n}}{\frac{n}{n} + \frac{\sqrt{n^2+4}}{\sqrt{n^2}}}$$

$$= \lim_{n\to\infty} \frac{5}{1+\sqrt{1+\frac{4}{n^2}}} = \boxed{\frac{5}{2}}$$

pos, finite

Since $\sum b_n$ diverges

$\Rightarrow \sum a_n$ diverges by L.C.T.

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28) $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$

$$\lim_{n\to\infty} \frac{\overbrace{\tan\left(\frac{1}{n}\right)}^{a_n}}{\underbrace{\frac{1}{n}}_{b_n}}$$

$\frac{1}{n} \to$ Divergent Harmonic Series.

~~L'Hop~~

$$\lim_{n\to\infty} \frac{\sec^2\left(\frac{1}{n}\right)\cdot\left(-n^{-2}\right)}{\left(-n^{-2}\right)}$$

$$= \frac{1}{\cos^2\left(\frac{1}{\infty}\right)} = 1$$

Pos, finite

Since $\Sigma\, b_n$ diverges

$\Rightarrow \Sigma\, a_n$ diverges by L.C.T.